Valuing Plug-In Hybrid Electric Vehicles’ Battery Capacity Using a Real Options Framework

Derek M. Lemoine*

Plug-in hybrid electric vehicles (PHEVs) enable their drivers to choose whether to use electricity or gasoline, but this fuel flexibility benefit requires the purchase of additional battery capacity relative to most other vehicles. We value the fuel flexibility of PHEVs by representing the purchase of the battery as the purchase of a strip of call options on the price of transportation. We use a Kalman filter to obtain maximum likelihood estimates for three gasoline price models applied to a U.S. municipal market. We find that using a real options approach instead of a discounted cash flow analysis does not raise the retail price at which the battery pays for itself by more than $50/kWh (or by more than 15%). A discounted cash flow approach often provides a good approximation for PHEV value in our application, but real options approaches to valuing PHEVs’ battery capacity or role in climate policy may be crucial for other analyses.

1. INTRODUCTION

By enabling low-carbon, domestic sources of electricity to provide energy for transportation, plug-in hybrid electric vehicles (PHEVs) could provide the triple benefit of reducing greenhouse gas emissions, lessening economic dependence on petroleum, and improving urban air quality, all while maintaining the range and convenience of gasoline-powered conventional vehicles. Like all-electric vehicles, PHEVs enable low-carbon sources of electricity to provide motive power and could displace transportation sector pollutant emissions to centralized sources where they are more easily controlled and where they are farther from population centers. Like conventional gasoline-fueled vehicles, a PHEV’s range is not limited by either its battery capacity or the time available to recharge its batteries. A PHEV, in a sense, has two fuel tanks, and its owner must decide how to use them: the vehicle can use electricity stored from the grid to travel some number of miles in charge-depleting “blended” or “electric

*Ph.D. candidate, Energy and Resources Group, 310 Barrows Hall, University of California, Berkeley, CA 94720-3050, dlemoine@berkeley.edu.
vehicle” modes before switching to liquid-fueled “charge-sustaining” mode, or it can run immediately in charge-sustaining mode by combusting liquid fuels such as gasoline, diesel, or ethanol. Hybrid electric vehicles (HEVs) like the Toyota Prius run in charge-sustaining mode all the time, never using electricity from the grid. Because electricity is cheap compared to gasoline, PHEVs can substantially lower fuel costs even when compared to HEVs. However, because charge-depleting mode requires the battery to store more energy than is needed for charge-sustaining mode, PHEV batteries must have greater capacity than HEV batteries. Deciding whether to purchase a PHEV therefore involves, among many other factors, weighing a tradeoff between the cost of the additional battery capacity and the fuel savings it provides. We develop and apply new methods that value the fuel flexibility offered by a PHEV’s battery. These methods represent the purchase of a PHEV as the purchase of a strip of call options based on the price of liquid fuels. Moving beyond discounted cash flow analyses, we show how accounting for the uncertain path of future fuel prices and for the PHEV driver’s ability to respond to these prices by choosing the day’s fuel can raise the battery price at which PHEVs pay for themselves.

PHEVs may operate in three energy management modes (Bradley and Frank, 2009), and there are three main strategies for determining which energy management mode to employ at a given time and so how best to use grid-supplied electricity (Gonder and Markel, 2009). In charge-sustaining mode, the battery’s state of charge does not vary much and is frequently replenished by the internal combustion engine or by regenerative braking. Liquid fuels are the net source of energy, with the electric motor used to assist the engine or to power the vehicle when idling or at low speed. This mode is already familiar from HEVs. The other two modes are both charge-depleting. In blended mode, the battery provides a portion of the net source of energy (i.e., the vehicle uses grid-supplied electricity) and the engine supplies the rest of the energy. In electric vehicle mode, the battery is the entire net source of energy because the engine does not turn on. Blended and electric vehicle modes represent two different ways of using electricity stored from the previous charging session.1

The three primary energy management strategies differ in how to use the stored electrical energy, but all run the vehicle in charge-sustaining mode once the battery’s state of charge drops low enough and all continue to operate in charge-sustaining mode until the next recharging. An all-electric range (AER)-focused strategy operates the PHEV in electric vehicle mode as long as the state of charge is sufficiently high. A PHEVx is a vehicle that uses this AER-focused strategy and can travel x miles in electric vehicle mode on a reference driving

1. We use “charge-depleting mode” to include both the electric vehicle mode and the blended mode, but Bradley and Frank (2009) use “charge-depleting” mode only to refer to what we call the blended mode.
cycle. In contrast, a blended PHEV uses either an engine-dominant or an electric-dominant blended strategy. In the engine-dominant blended strategy, the vehicle maximizes its system efficiency in blended mode by using its electric motor only for low speeds and for keeping the engine within a defined operating range. In the electric-dominant blended strategy, the electric motor is the dominant power source in blended mode and the engine only turns on when the power demands exceed the capability of the electric motor. The engine-dominant blended strategy can maximize efficiency for a given driving distance, but the strategy’s optimality is not robust to changes in this distance because shorter-than-planned trips favor AER-focused operation (O’Keefe and Markel, 2006). However, the AER-focused strategy requires more expensive electric components because they must be able to meet the highest power demands of electric vehicle mode. Therefore the electric-dominant blended strategy may represent the best compromise between cost, fuel efficiency, and robustness to driving profile (Gonder and Markel, 2007). We do not yet know what design decisions or usage patterns PHEVs will show once they are commonly marketed. For an overview of the state of PHEV development, deployment, and research, see Bradley and Frank (2009).

While PHEVs’ additional battery capacity should increase their manufacturing cost relative to comparable conventional vehicles (CVs) and HEVs, their ability to use grid-supplied electricity can result in significant fuel savings via efficient electric operation. These fuel savings have typically been valued by treating them as a series of future cash flows which are discounted back to the present. Lemoine et al. (2008) find that with recent U.S. gasoline and electricity prices, PHEV owners would generally want to run their vehicles on grid-supplied electricity, and they would often want to do so even under real-time electricity pricing. If adopted, PHEVs could therefore provide many of the benefits of all-electric vehicles because their owners would indeed want to use grid-supplied electricity. However, Lemoine et al. also find that unless batteries become much cheaper or gasoline consistently much more expensive, the present value of expected fuel savings with vehicle efficiencies as in EPRI (2002) would probably not offset the cost of the additional battery capacity that a PHEV would require. Markel and Simpson (2006) and Scott et al. (2007) use similar valuation methods and also reach pessimistic conclusions about the near-term economics of PHEV adoption. These conclusions are most pessimistic when PHEVs are compared to HEVs because HEVs can provide many of the efficiency gains relative to CVs but with a much smaller battery. While vehicle purchasers base their decisions on many factors other than fuel savings (e.g., Heffner et al.,

2. Gonder et al. (2007) show that when driven on real driving cycles, which have greater power demands than the usual reference cycles, PHEVs may not achieve their AER. Gonder and Simpson (2007) propose to redefine the all-electric range (and the x in PHEVx) in terms of displaced petroleum consumption so that it is not specific to PHEVs using electric vehicle mode. Also, some use PHEVx to refer broadly to a PHEV with x miles of charge-depleting range, whether all-electric or blended. Gonder and Markel (2007) suggest that the overall best strategy may be one in which the vehicle predicts the driving cycle and so can switch between electric-dominant and engine-dominant blended strategies depending on expected trip distance.
2007), assessing the value of fuel savings is important because if the additional battery capacity does pay for itself, that should speed adoption by individual and, especially, fleet purchasers.

By using expected fuel prices to value PHEVs, all three previous analyses ignore the flexibility that PHEV owners have regarding whether to charge their vehicles from the grid. Gasoline prices are unpredictable, and their volatility may confer value on the ability to select which fuel to use. The previous assessments value a PHEV exactly like a nearly identical vehicle that requires its owner to use electricity for each day's initial miles. A PHEV should be (weakly) more valuable than this fictional vehicle precisely because its owner can choose which fuel to use for its initial miles of operation. On a given day, if the cost per mile of PHEV operation using grid-supplied electricity is less than the cost per mile of PHEV operation in gasoline-fueled charge-sustaining mode, then the vehicle owner can pay the least by charging the PHEV from the grid. However, if the cost per mile of operation is less for charge-sustaining mode, then the PHEV owner can forgo charging and use gasoline for the entire trip. The PHEV's battery provides fuel savings relative to an HEV if electricity is cheaper than gasoline on a cost per mile basis, and it produces neither savings nor additional fuel costs if electricity is more expensive than gasoline. Each day's fuel savings payoff is therefore nonlinear because the owner's option to forgo charging means that the fuel savings should never be negative. Using expected fuel prices to estimate fuel savings undervalues the battery capacity because, in fact, the possible gains (fuel savings) from the battery purchase are unlimited while the possible losses (negative fuel savings) are limited by the ability to use gasoline instead of grid-supplied electricity (Figure 1).

We extend the previous PHEV analyses by treating the purchase of a PHEV as the purchase of a strip (or bundle) of European call options on the price of transportation. Ignoring non-fuel factors such as end-of-life battery value, the cost of battery replacement, vehicle-to-grid electricity sales as a revenue opportunity, and non-financial benefits of PHEV ownership, the purchase of a PHEV's battery has the same payoff as the purchase of a strip of European call options in which the payoffs are the fuel savings in dollars per mile and each option in the strip corresponds to the decision on some day \( d \) about whether to use grid electricity for some mile \( m \) of PHEV travel. Using grid-supplied electricity for mile \( m \) on day \( d \) means that the PHEV owner is exercising that call option bought through the PHEV's battery capacity, and forgoing grid-supplied electricity for liquid fuels means she is allowing that particular option to expire. Because they have the same option value relative to an all-electric vehicle. An all-electric vehicle can have option value relative to a conventional vehicle if it is an additional vehicle in a household or fleet. In that case, the dual-fuel capability comes from the household or fleet operator choosing what type of vehicle to use rather than choosing in which mode to run a single vehicle (thanks to Jason Wolf for this point).

4. A PHEV also has option value relative to an all-electric vehicle. An all-electric vehicle can have option value relative to a conventional vehicle if it is an additional vehicle in a household or fleet. In that case, the dual-fuel capability comes from the household or fleet operator choosing what type of vehicle to use rather than choosing in which mode to run a single vehicle (thanks to Jason Wolf for this point).

5. A European call option confers the right, but not the obligation, to buy a predefined asset at a particular price at a predefined time. An American call option differs in that it may be exercised at any time up to and including the time of expiration.
Figure 1. The Expected Payoff from One Day's Charging Option Can Depend on the Flexibility of the Charging Strategy

The expected fuel price completely determines the expected value of the simplistic strategy in which the PHEV owner always charges from the grid. As long as there is any positive probability of gasoline prices being low enough (or electricity prices being high enough) to make a PHEV owner want to forgo charging, then the expected value of the resulting charging strategy must be greater than the expected value of the simplistic charging strategy because it has a greater payoff in some states of the world and the same payoff in all other states of the world. Using expected fuel prices undervalues the more sophisticated charging strategy.

The expected savings if owner always charges is the PHEV's battery and this strip of options also have the same value. Extending previous results, we value the options under various assumptions about the stochastic processes that electricity and gasoline prices follow. These same techniques could be used to value the savings of any dual-fuel vehicle, including gasoline-ethanol flex-fuel vehicles, relative to a single-fuel vehicle. Our methods appreciate drivers' opportunities to choose which fuel to use based on relative prices, and they may make PHEVs seem more economically attractive to both expenditure-sensitive vehicle buyers and carbon-sensitive governments than have the discounted cash flow analyses to date.

Our work is similar to the increasingly popular real options approach to valuing electricity generation assets as a strip of spread options between the price of electricity and the cost of the natural gas required to generate a unit of electricity (e.g., Deng et al., 2001). There have been other attempts to value energy-saving technologies as strips of options. Sezgen et al. (2007) value three demand-response technologies for reducing electricity use by treating them as combinations of options. Tareen et al. (2000) and Vedenov et al. (2006) use a real options approach to determine the price thresholds at which wholesale fuel purchasers should shift, respectively, from petroleum diesel to biodiesel or from conventional gasoline to ethanol blends. They model the fuel spot prices as following geometric Brownian motion, which is a type of price process.
commonly used to represent stock prices. However, as we will discuss below, geometric Brownian motion is generally inappropriate for modeling commodity price processes, including electricity and gasoline price processes.

We first formally represent the purchase of a PHEV’s battery capacity as the purchase of a strip of options. We then describe the mean-reverting price processes we use to represent the evolution of gasoline and electricity spot prices, with the appendices providing the closed-form solutions for pricing the options conferred by PHEVs and other dual-fuel vehicles. We next estimate the real options value of a PHEV20 purchased in Sacramento, California in May of 2009. Finally, we discuss the implications of our results for vehicle purchase economics, for business models, and for greenhouse gas abatement.

2. REPLICATING PHEV FUEL SAVINGS BY A STRIP OF OPTIONS

Because a PHEV’s initial miles may use grid-supplied electricity or gasoline, a PHEV purchaser buys the option to use whichever fuel is cheapest so long as he has sufficient battery capacity to do so. Relative to an HEV with the same efficiency in charge-sustaining mode, the value of the PHEV’s additional battery capacity—and the maximum premium a prospective vehicle purchaser may be willing to pay for the PHEV—is the value of this right to choose the cheapest fuel, which in a world of certain fuel prices would equal the present value of the fuel savings at these prices (as in Markel and Simpson, 2006; Scott et al., 2007; Lemoine et al., 2008). However, the battery capacity determines not just the willingness to pay but also the incremental cost of the PHEV (cf. Lipman and Delucchi, 2006 for HEVs; Lemoine et al., 2008). The break-even battery price for a PHEV relative to an HEV is the dollars per kilowatt-hour (kWh) of battery capacity that would end up costing the PHEV owner the full value of the additional battery capacity. If the additional battery value is $5000 for a PHEV that requires 5 kWh more battery capacity relative to an HEV, the break-even battery price is $1000/kWh. The break-even battery price for a PHEV relative to a conventional vehicle must consider not just the flexibility gained from charge-depleting mode but also the efficiency gains from charge-sustaining mode. The thusly calculated break-even battery price is increased by assuming that batteries comprise the entire incremental vehicle cost and do not degrade, but it is decreased by ignoring the possible scrap value of the batteries and the possibility of using the PHEV’s battery to sell electricity back to the grid. We use the difference between actual battery costs and break-even battery costs as a proxy for the attractiveness of PHEVs to vehicle buyers.

We replicate the fuel cost savings of a PHEV by the payoff of a strip of options corresponding to each mile of a PHEV’s battery capacity. Because purchasing the strip of options would be financially equivalent to purchasing the fuel flexibility of a PHEV, we treat the value of the strip of options as equal to the value of the PHEV’s fuel flexibility. One mile of battery capacity is the kWh of battery capacity needed to drive the vehicle 1 mile using stored grid
electricity (i.e., in charge-depleting mode) on a reference driving cycle. For each mile of travel to be used on a given day up to the total miles of battery capacity, the PHEV owner may draw the corresponding amount of electricity from the grid if electricity is cheap enough relative to gasoline or may drive that mile in gasoline-fueled charge-sustaining mode if not.\textsuperscript{5} The value of having that mile of battery capacity on day $d$ is therefore the greater of 0 and the savings from using electricity rather than only gasoline to drive that mile. At the time of the vehicle purchase, this payoff can be replicated by purchasing a European call option expiring on day $d$. As with the real options approach to valuing electricity generation assets, the value of that mile of battery capacity is then the sum of the call options expiring on each day of the battery’s life, with each day’s option weighted by the mile’s likelihood of being needed on that day. Summing the value of each of the miles of battery capacity then yields the maximum amount a vehicle buyer ought to be willing to pay for the battery on the basis of the fuel flexibility it provides.

If the price of electricity is deterministic, then the call option expiring on day $d$ is a vanilla European call option whose strike price is the cost of the grid-supplied electricity (and, for blended PHEVs, the gasoline) needed to travel one mile in charge-depleting mode. If the price of electricity is stochastic, then we have a European spark spread call option. The term “spark spread” generally refers to the gross income available from combusting natural gas to make electricity, and its magnitude depends on the efficiency of combustion and the prices of electricity and gas. In their application to natural gas-fired electricity generation assets, spark spread call options proxy the profit from converting natural gas into electricity by having a payoff that is the greater of zero (reflecting the ability to choose not to run the plant at a given time) and the price of electricity (which is the revenue from producing one unit of electricity) minus the product of a prespecified heat rate and the price of natural gas (which ideally gives the cost to the generator of producing one unit of electricity).\textsuperscript{7} In our case, the spark spread call option proxies the fuel savings from using electricity instead of gasoline by having a payoff that is the greater of zero and the cost of the gasoline required to travel one mile in charge-sustaining mode minus the cost of the electricity (and, for blended PHEVs, the gasoline) required to travel one mile in charge-depleting mode. This option fits the typical form of a spark spread better if we specify the payoffs in dollars per gallon rather than in dollars per mile, for then the analogue of the heat rate is the kWh/gallon resulting from the gasoline-fueled efficiency and the charge-depleting efficiency.

Proceeding more formally, we first consider a PHEV$x$. Let $P_g(t)$ be the retail price of gasoline in $/gallon at time $t$, $P_e(t)$ be the retail price of electricity in $/kWh at time $t$, $\eta_{cs}$ be the efficiency in miles/gallon of the PHEV in charge-

\textsuperscript{5} We focus on gasoline as the fuel source in competition with electricity, but our methods could be applied to another liquid fuel such as ethanol or diesel.

\textsuperscript{7} See Deng et al. (2001), Carmona and Durrleman (2003), and Eydeland and Wolyniec (2003) for more on spark spread options.
sustaining mode, and $\eta_e$ be the efficiency in miles/kWh of the PHEV in electric vehicle mode. $K_V$ is the strike price of a vanilla European call option, and $K_H$ is the "heat rate" in a European spark spread call option. Let $d$ be the number of days in the life of the vehicle. We assume that charging occurs only once per day. The value of the battery capacity required to travel mile $m$ is:

$$V_m = \sum_{d=1}^{\bar{d}} C_d(0) p_m(d)$$

(1)

where $p_m(d)$ is the probability of driving mile $m$ on day $d$ and $C_d(0)$ is the value at the time of purchase of the call option expiring on day $d$ with the following payoff on that day:

$$C_d(d) = \frac{1}{\eta_{cs}} \max\left(P_g(d) - \frac{\eta_{cs}}{\eta_e} P_e(d), 0\right)$$

(2)

If retail electricity prices are deterministic, we have a vanilla European call option with strike price

$$K_V(d) = \frac{\eta_{cs}}{\eta_e} P_e(d),$$

and if retail electricity prices vary stochastically as, for instance, in real-time electricity pricing, we have a spark spread call option with heat rate $K_H = \frac{\eta_{cs}}{\eta_e}$.

We next consider a blended PHEV. Let $\eta_{cd,g}$ be the miles per gallon achieved in charge-depleting blended mode and $\eta_{cd,e}$ be the miles per kWh achieved in charge-depleting blended mode. By including the gasoline used in charge-depleting mode and rearranging, we now have:

$$C_d(d) = \left(\frac{1}{\eta_{cs}} - \frac{1}{\eta_{cd,g}}\right) \max\left(P_g(d) - \frac{\eta_{cs} \eta_{cd,g}}{\eta_{cd,e} (\eta_{cd,g} - \eta_{cs})} P_e(d), 0\right)$$

(3)

If retail electricity prices are deterministic, we have a vanilla European call option with strike price

$$K_V(d) = \frac{\eta_{cs} \eta_{cd,g}}{\eta_{cd,e} (\eta_{cd,g} - \eta_{cs})} P_e(d),$$

and if retail electricity prices vary stochastically, we have a spark spread call option with heat rate

$$K_H = \frac{\eta_{cs} \eta_{cd,g}}{\eta_{cd,e} (\eta_{cd,g} - \eta_{cs})}.$$
Once we have the value of each mile of battery capacity from equation (1) using the closed-form solutions for \( \bar{C}_d(0) \) given in the appendices, we can value the fuel savings of the PHEV relative to a comparable HEV or CV. Let \( \bar{m} \) be the number of miles that the PHEVx can travel in charge-depleting electric vehicle mode or the blended PHEV can travel in charge-depleting blended mode, let \( \eta_{\text{comp}} \) be the fuel efficiency of the comparable vehicle in miles/gallon, and let \( B_{\text{phev}} \) and \( B_{\text{comp}} \) represent the nominal battery capacity in kWh of the PHEV and the comparable vehicle. Depth-of-discharge limitations mean that the PHEVx’s nominal battery capacity should be greater than would be found by dividing its all-electric range by the charge-depleting mode efficiency. Since CVs’ lead-acid batteries have a very different cost structure, we can approximate a CV by setting \( B_{\text{comp}} = 0 \). The total value of the PHEV’s battery capacity relative to a comparable vehicle is:

\[
V_{\text{phev}} = PV_{\text{cs}} + \sum_{m=1}^{\bar{m}} V_m
\]  

(4)

where \( V_m \) is defined in equation (1) as the value of the battery capacity required to travel mile \( m \) in electric vehicle or blended mode and \( PV_{\text{cs}} \) is the present value of the expected fuel savings obtained by the change in gasoline-fueled vehicle efficiency from \( \eta_{\text{comp}} \) to \( \eta_{\text{cs}} \) if charge-sustaining mode is used in all the miles of driving. If we are comparing a PHEV to an HEV with the same efficiency in charge-sustaining mode, then \( PV_{\text{cs}} = 0 \). The break-even battery cost for the PHEV relative to the comparable CV or HEV is:

\[
BC_{\text{phev}} = \frac{V_{\text{phev}}}{B_{\text{phev}} - B_{\text{comp}}}
\]  

(5)

3. GASOLINE AND ELECTRICITY PRICE PROCESSES

The value of each call option at the time of purchase \( \bar{C}_d(0) \) depends on beliefs about the future evolution of gasoline and electricity prices. We model retail gasoline prices in three different ways and provide closed-form option pricing solutions under deterministic electricity prices in Appendix 1. When both electricity and gasoline prices are stochastic, we model them each as following one-factor mean-reverting price processes, resulting in the option pricing formula in Appendix 2. In the next section, we estimate option value using recent gasoline prices and assuming deterministic, constant electricity prices.

8. If \( p_m(d) \) is constant, we have \( V_m = \sum_{d=1}^{\bar{d}} C_d(0) p_m \) and so \( \sum_{m=1}^{\bar{m}} V_m = \sum_{d=1}^{\bar{d}} C_d(0) \sum_{m=1}^{\bar{m}} p_m \). Since \( p_m \) is the probability that at least \( m \) miles of charge-depleting range are needed, and since \( m \) is non-negative and integer-valued, we have by the tail sum formula that \( \sum_{m=1}^{\bar{m}} p_m = \hat{m} \), where \( \hat{m} \) is the expected number of miles per day that could have been driven using grid-supplied electricity (so \( \hat{m} \leq \bar{m} \)). Thus \( \sum_{m=1}^{\bar{m}} V_m = \hat{m} \sum_{d=1}^{\bar{d}} C_d(0) \) if \( p_m(d) \) is a constant for each mile \( m \).
Gasoline price fluctuations are linked to fluctuations in the price of petroleum, but they are also affected by fluctuations in refinery capacity, refinery product mix, air quality regulation, and aggregate driving demand. Retail electricity price fluctuations depend primarily on the institutional factors underlying retail electricity rates and secondarily on the grid’s structure, pollutant regulations, and fuel prices (especially natural gas in the U.S.). While equity prices need not have any natural level to which they should revert and so can be modeled using geometric Brownian motion, commodity prices are driven by supply of and demand for the underlying commodity. They should revert to their long-run marginal cost trend and so should not be modeled by geometric Brownian motion (cf. Schwartz, 1997; Baker et al., 1998; Pindyck, 2001). Reinforcing this logic, Mazaheri (1999) finds that the convenience yield in gasoline follows a non-stationary mean-reverting long-term process and attributes this to the market expecting mean reversion in spot prices. We will therefore use only mean-reverting models to approximate the path of gasoline spot prices. In addition, we only use constant volatility models for greater tractability in pricing options.\(^9\)

The first model we use for retail gasoline prices is the one-factor model of Schwartz (1997), which represents the commodity spot price process as:

\[
dP_t = \kappa(\mu - \ln P_t)P_t dt + \sigma P_t dz
\]

where \(dz\) is an increment of standard Brownian motion and \(P_t\) is the spot price at time \(t\). The speed of mean reversion is determined by \(\kappa > 0\), and the long-term mean of the spot price is \(\mu\). By Itô’s Lemma, the process for the log of the spot price is therefore an arithmetic Ornstein-Uhlenbeck process:

\[
d(\ln P_t) = \kappa(\alpha - \ln P_t) dt + \sigma dz
\]

The long-term mean of the log of the gasoline spot price is given by \(\alpha = \mu - \frac{\sigma^2}{2\kappa}\).

When the log price exceeds (falls short of) \(\alpha\), its deterministic trend goes negative (positive), driving it back towards \(\alpha\).

The Schwartz and Smith (2000) non-stationary two-factor model extends the one-factor model. It decomposes the log of the spot price into \(\chi\), the short-term deviation from the equilibrium price level, and \(\xi\), the log of the equilibrium long-term price level. Unlike in the Schwartz (1997) one-factor model, the long-term price level is now itself stochastic, its evolution representing the result of long-lasting fundamental changes as opposed to the short-term variations that

9. In reality, gasoline price volatility exhibits clustering and mean-reverting behavior. It would therefore be more realistic to treat the volatility parameter as following its own stochastic process. Lee and Zyren (2007) argue that GARCH models are good for estimating the volatility of petroleum and petroleum product spot prices, and Sadorsky (2006) concludes that GARCH(1,1) models work well for estimating volatility in crude oil and gasoline futures returns.
drive $\chi_t$. Formally, we have four equations, with the first from the presentation of Korn (2005):

\begin{align}
\ln P_t &= \chi_t + \xi_t - \frac{\mu_\xi}{\kappa} \\
\dot{\chi}_t &= -\kappa \chi_t + \sigma_\chi dt + \sigma_\xi d\xi_t \\
\dot{\xi}_t &= \mu_\xi dt + \sigma_\xi d\xi_t \\
\dot{\xi}_t d\xi_t &= \rho_{\xi,\xi} dt
\end{align}

(8) describes the composition of log prices, (9) describes the mean-reverting short-term variations, (10) describes the evolution of the equilibrium price level, and (11) describes the correlation between the two Brownian motions. The short-term deviations revert towards zero with speed determined by $\kappa$ in an exponential Ornstein-Uhlenbeck process, and the long-term equilibrium price level follows a geometric Brownian motion process with expected change determined by $\mu_\xi$. These four equations lead to the following stochastic differential equation (as in Korn, 2005):

\begin{equation}
\frac{d}{dt}(\ln P_t) = \kappa (\xi_t - \ln P_t) dt + \sigma_S d\xi_t
\end{equation}

where

\begin{align}
\sigma_S &= \sqrt{\sigma_\chi^2 + \sigma_\xi^2 + 2\rho_{\chi,\xi}\sigma_\chi\sigma_\xi} \\
d\xi_t &= (\sigma_\chi d\xi_t + \sigma_\xi d\xi_t)/\sigma_S
\end{align}

This model captures uncertainty about the level of the long-term equilibrium price as well as the fact that it too can evolve, leading Pindyck (2001) to support it on logical and empirical grounds. It also is equivalent to some models which represent the price process using a stochastic convenience yield (Schwartz and Smith, 2000).

Finally, Korn (2005) extends the Schwartz and Smith (2000) non-stationary two-factor model to include mean reversion in the long-term equilibrium price. This allows the overall model to be stationary. He redefines $\ln P_t$ and $d\xi_t$ as follows:

\begin{align}
\ln P_t &= \chi_t + \frac{\kappa}{\kappa - \gamma} \xi_t - \frac{\gamma - \kappa}{\kappa - \gamma} \Theta \\
\dot{\xi}_t &= \gamma (\Theta - \xi_t) dt + \sigma_\xi d\xi_t
\end{align}
\( \Theta \) is the mean of the distribution of the long-term log price level, and the long-term log price process reverts to the mean level \( \Theta \) with speed given by \( \gamma \). As Korn notes, we can derive a version of the Schwartz (1997) model with \( \xi = \Theta \) and a version of the Schwartz and Smith (2000) model if we take the limit as \( \gamma \to 0 \) and \( \gamma \Theta \to \mu \). Combining (13) and (14) with (9) and (11), we have:

\[
d(ln P_t) = \kappa (\xi_t - ln P_t) dt + \sigma S d z_S
\]

where

\[
\sigma_S = \sqrt{\frac{\kappa^2}{\kappa - \gamma} + \frac{\kappa^2}{(\kappa - \gamma)^2} \sigma_x^2 + 2 \frac{\kappa}{\kappa - \gamma} \rho x \xi \sigma_x \sigma_x}
\]

\[
d z_S = \left( \frac{\kappa}{\kappa - \gamma} \sigma_x d z_x + \sigma_x d z_x \right) / \sigma_S
\]

After estimating the parameters for crude oil in each of these three models, Korn finds that the two-factor models both outperform the one-factor model and that his stationary two-factor model has some advantages in pricing long maturity contracts and no disadvantages relative to the non-stationary two-factor model of Schwartz and Smith (2000).

It is not unreasonable to approximate near-term retail electricity prices as deterministic. If PHEV owners buy electricity based on a preset tariff as do most residential U.S. electricity customers, their rates do not change from day to day unless their aggregate consumption over a recent timespan pushes them into a higher rate block. Their rate structure only changes whenever the utility alters its tariffs, which may require approval from regulatory bodies. These consumers experience nearly constant (and basically predictable) electricity prices with jumps whose arrival times are measured in months or years.

In contrast, when PHEV owners use real-time electricity pricing, they are exposed to wholesale price variations on a sub-daily timescale. This pricing system requires a stochastic treatment of electricity prices. Knittel and Roberts (2005) find evidence of mean reversion in California's hourly spot prices from 1998–2000. Huisman et al. (2007) argue that the right model for electricity day-ahead markets uses a panel data approach in which each hour's price follows its own stochastic process. They find that hourly prices are mean-reverting, and after estimating the parameters, they find that mean reversion is greater in off-peak hours. We can therefore justify representing an hour's electricity prices as a one-factor mean-reverting process when assuming PHEV owners are subject to real-

10. It is possible that the Schwartz and Smith (2000) two-factor model will outperform the stationary two-factor model of Korn (2005) in the future if petroleum scarcity rents increase over time due to structural shifts in supply such as "peak oil" or in demand such as the continuing rise of emerging economies.
time pricing (see Appendix 2 for more information). The pricing formula with two mean-reverting fuel price processes could be applied to any dual-fuel vehicle, including flex-fuel vehicles that can use both gasoline and high-percentage ethanol blends.

**Empirical Application: A Prospective PHEV20 Purchase in Sacramento, California on May 6, 2009**

We now estimate the value of a hypothetical PHEV with 20-mile all-electric range considered for purchase in Sacramento, CA on May 6, 2009, with the other vehicle option being an HEV with the same charge-sustaining efficiency. We parameterize each of the three gasoline price process models to value the PHEV20 for a vehicle purchaser who believes that gasoline prices over the 12-year vehicle lifetime are best forecast by the data-generating process operative over the previous decade. The Oil Price Information Service provides the daily municipal-average retail price for unleaded gasoline in Sacramento dating back to January 4, 1999, as reported by surveyed gasoline station owners. On the date of vehicle evaluation, the gasoline price was $2.405/gallon. Figure 2 shows the observed history of gasoline prices, and Table 1 provides summary statistics. An augmented Dickey-Fuller test fails to reject the null hypothesis of a unit root, while a Kwiatkowski-Phillips-Schmidt-Shin test rejects the null hypothesis of stationarity at the 1% significance level. Taken alone, these results favor non-stationary models such as that of Schwartz and Smith (2000) as descriptions of past retail gasoline prices.

We use maximum likelihood estimation to parameterize each price process model (Table 2). We obtain our estimate for the one-factor model by using the exact transition density (Phillips and Yu, 2007), and the standard errors come from the outer product estimate of the information matrix (Hamilton, 1994: 143). Estimation of the two-factor models requires use of the Kalman filter because \( \gamma \) and \( \xi \) are unobservable. Instead, all we observe are the retail gasoline prices \( P_t \), which are related to the state variables \( \gamma_t \) and \( \xi_t \) via equations (8) and (13) plus normally distributed and unknown measurement error \( \varepsilon \). This measurement error can represent variation in the accuracy of calculated average prices or variations in the match between the driver's realized price and the city-wide average price.

Schwartz and Smith (2000) and Korn (2005) each provide further details on

1. Because electricity is not readily storable over even a time frame of minutes and must obey physical constraints on transmission, wholesale electricity prices are better modeled by including spikes (Deng, 2000; Barlow, 2002; Eydeland and Wolyniec, 2003). However, spikes greatly complicate option pricing and, whether by preference or incentives, PHEV owners will more than likely charge at off-peak hours when spikes are less important and less likely. At a minimum, PHEV owners will be discouraged from charging during the high-demand times that can create a price spike.

12. The augmented DF test results in a test statistic of -2.45 while the KPSS test gives a test statistic of 1.58. The critical values at the 5% level are -2.86 for the DF test and 0.146 for the KPSS test. Both tests use 6 lags, as determined by an automatic bandwidth selection procedure. The augmented DF test includes a constant but not a time trend, and the KPSS test uses the quadratic spectral kernel.
Municipal-average retail price history for unleaded gasoline in Sacramento, California from January 4, 1999 through May 6, 2009. Also, for each of the three gasoline price models, the maximum likelihood forecasts for the ensuing 12 years along with their interdecile ranges. The interdecile range comes from the estimated volatility parameters in each model. It should not be read as a prediction interval because it does not include either the standard errors in the parameter estimation or the uncertainty about model specification. The dashed lines show the gasoline price ($1.21/gal) at which the owner of a PHEV with parameters as in Table 3 is indifferent between using gasoline or electricity.

Table 1. Descriptive Statistics for the Retail Gasoline Price Time Series (in $/gal)

<table>
<thead>
<tr>
<th>Observations (days)</th>
<th>3544</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2.249</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.7248</td>
</tr>
<tr>
<td>Minimum</td>
<td>1.087</td>
</tr>
<tr>
<td>Maximum</td>
<td>4.579</td>
</tr>
</tbody>
</table>

Data give each day's average for the Sacramento, California municipal area as reported by surveyed gasoline station owners. The time series extends from January 4, 1999 through May 6, 2009.

deriving the transition equations. Harvey (1989) describes the Kalman filter, and we follow his recommendations for initialization of the state variables and his analytical derivation of the score vector and information matrix. We maximize the log likelihood by using the method of steepest ascent with a variety of starting values (Hamilton, 1994: 137).
Table 2. Maximum Likelihood Estimates and Standard Errors for the Three Gasoline Price Models' Parameters Based on the Previous 10 Years' Prices

<table>
<thead>
<tr>
<th>Parameter</th>
<th>One-factor</th>
<th>Two-factor non-stationary</th>
<th>Two-factor stationary</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>ln($/gal)</td>
<td>1.034 (1.974x10^3)</td>
<td>-</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>ln($/gal)</td>
<td>-</td>
<td>7.876x10^{-1} (1.525x10^3)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>ln($/gal) day^{-1}</td>
<td>-</td>
<td>1.604x10^{-5} (9.629x10^{-5})</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>day^{-1}</td>
<td>7.988x10^{-4} (3.285x10^{-4})</td>
<td>5.004x10^{-1} (7.736x10^{-1})</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>day^{-1}</td>
<td>-</td>
<td>4.077x10^{-4} (2.762x10^{-4})</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>ln($/gal) day^{-1/2}</td>
<td>5.700x10^{-3} (2.898x10^{-3})</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>ln($/gal) day^{-1/2}</td>
<td>-</td>
<td>9.048x10^{-5} (8.615x10^{-5})</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>ln($/gal) day^{-1/2}</td>
<td>-</td>
<td>5.725x10^{-3} (1.523x10^{-4})</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>-</td>
<td>4.983x10^{-2} (6.094x10^{-2})</td>
<td>-</td>
</tr>
<tr>
<td>$\chi_0$</td>
<td>ln($/gal)</td>
<td>-</td>
<td>7.473x10^{-6} (9.030x10^{-6})</td>
</tr>
<tr>
<td>$\xi_0$</td>
<td>ln($/gal)</td>
<td>-</td>
<td>8.776x10^{-4} (1.341x10^{-4})</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>ln($/gal)</td>
<td>-</td>
<td>9.926x10^{-5} (9.520x10^{-6})</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>1.327x10^4</td>
<td>1.326x10^4</td>
<td>1.326x10^4</td>
</tr>
<tr>
<td>AIC</td>
<td>2.654x10^4</td>
<td>2.651x10^4</td>
<td>2.651x10^4</td>
</tr>
<tr>
<td>SIC</td>
<td>2.653x10^4</td>
<td>2.650x10^4</td>
<td>2.650x10^4</td>
</tr>
</tbody>
</table>

$\epsilon$ is measurement error for the Kalman filter applications, and the other parameters are as defined in section 3. The bottom three rows show the maximized log likelihood, the Akaike information criterion (AIC), and the Schwarz (Bayesian) information criterion (SIC) for each model's application. Note that the one-factor model's estimation is exact whereas the two-factor models use an iterative procedure that concludes with sufficient convergence.

For each set of maximum likelihood estimates, the price at a future date is a lognormal distribution with mean and interdecile range as shown in Figure 2. The non-stationary two-factor model's price forecast and variance continue to grow through the 12-year vehicle lifetime and beyond, but both of the stationary models approach equilibrium distributions within the vehicle's lifetime. The Akaike information criterion and the Schwarz (Bayesian) information criterion both favor the one-factor model, but the one-factor result is exact while the two-factor models' results are defined by convergence, so the relatively small gap in likelihood and information scores may not stand if all results were exact.

We value the PHEV20 using the estimated processes for gasoline prices and the assumptions shown in Table 3. The May 2009 residential baseline electricity price was $0.0931/kWh, and we take it to be constant over the 12-year vehicle lifetime. Because we do not adjust it for risk, this constant electricity
price corresponds either to electricity prices that risklessly maintain the level of $0.0931/kWh over the 12-year time horizon or to electricity prices that are forecast to increase but have a risk-adjusted forecast of $0.0931/kWh for all dates within the 12-year time horizon. Vehicle efficiency and battery size parameters are for a mid-size vehicle and come from Samaras and Meisterling (2008), who in turn obtain them from EPRI (2001). We assume that the 6.7 kWh battery is not replaced, the vehicle is a PHEV20 driven its entire all-electric range every day for 12 years, the vehicle's battery does not have scrap value, and the risk-free interest rate is constant and certain at 1%/year. Note that providing sufficient calendar life and cycle life for batteries subject to frequent deep discharges is a key challenge for PHEVs (Burke, 2007), and the batteries may have scrap value if utilities choose to refurbish them for grid services.

Table 3. Assumed Vehicle Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{od}$</td>
<td>mi kWh$^{-1}$</td>
<td>3.45</td>
</tr>
<tr>
<td>$\eta_{es}$</td>
<td>mi gal$^{-1}$</td>
<td>45</td>
</tr>
<tr>
<td>$\eta_{comp}$</td>
<td>mi gal$^{-1}$</td>
<td>45</td>
</tr>
<tr>
<td>$B_{phev}$</td>
<td>kWh</td>
<td>6.7</td>
</tr>
<tr>
<td>$B_{comp}$</td>
<td>kWh</td>
<td>1.3</td>
</tr>
<tr>
<td>$P_s(0)$</td>
<td>$$/gal</td>
<td>2.405</td>
</tr>
<tr>
<td>$P_e$</td>
<td>$$/kWh$^{-1}$</td>
<td>0.0931</td>
</tr>
<tr>
<td>$r$</td>
<td>% yr$^{-1}$</td>
<td>1</td>
</tr>
</tbody>
</table>

In the two-factor models, we use the same price of risk for each source of volatility. The value of the PHEV in this real options model is the sum of three factors; the value of the PHEV if fuel prices were deterministic

Figure 3 shows the value of a PHEV20 for varying prices of risk for the gasoline price processes. We include a price of risk because we are in an incomplete market setting. The market price of risk determines the market value of the PHEV’s fuel flexibility, but individuals or corporations may use their own appetites for risk to determine their private valuations (Eydeland and Wolyniec, 2003: 437). We do not estimate a market price of risk because that parameter is not important for our analysis and because properly estimating it for an energy commodity requires use of forward prices in addition to spot prices (Kolos and Ronn, 2008). In the two-factor models, we use the same price of risk for each source of volatility. The value of the PHEV in this real options model is the sum of three factors: the value of the PHEV if fuel prices were deterministic

13. The overall efficiency of marketed PHEVs is difficult to predict. It is sensitive to vehicle design, vehicle type (e.g., EPRI, 2001, 2002), battery size (Shiau et al., 2009), energy management strategy (O’Keefe and Markel, 2006), driving profile (Gonder et al., 2007), and rider preferences such as air conditioner usage (Karner and Francfort, 2007).
(so the $\sigma$ terms equal 0), the additional value when fuel prices are uncertain but owners must charge their vehicles from the grid (this value does not equal the deterministic value because the gasoline price distributions are lognormal), and the additional value from considering the nonlinearity in payoffs that fuel flexibility produces. We define the equivalent discount rate for a price of risk as that rate (expressed for annual compounding) which produces the same value when applied to the uncertain world without fuel flexibility as does the price of risk (with a 1%/year risk-free interest rate) in a risk-adjusted world without fuel flexibility. We use the equivalent discount rate to calculate the PHEV's value in the deterministic world and we use the price of risk to calculate the value in an uncertain world with fuel flexibility.

In the deterministic world, the one-factor model values the PHEV most highly because, in the parameterization of Table 2, its long-term mean is greater than that of the stationary two-factor model and because its forecast increases from the initial gasoline price more quickly than does the non-stationary two-factor model's forecast. The non-stationary two-factor model's wider interdecile range in its forecasts (as shown in Figure 2) means that it obtains greater incremental value from considering the lognormality of the future gasoline prices, because its expected price is that much farther from its expected log price. The incremental value from uncertainty is trivial in the estimated stationary two-factor model because its narrow lognormal distribution is almost normal. The non-stationary two-factor model's interdecile range comes closest to the critical gasoline price of $1.21/gallon (where the PHEV owner is indifferent between using grid-supplied electricity or gasoline), giving it the greatest incremental value from fuel flexibility.

The opportunity to forgo charging is the source of fuel flexibility's incremental value, but Figure 4, a Monte Carlo simulation of 1000 vehicle lifetimes for each price process, shows that it is rare to forgo charging in any price model other than the non-stationary two-factor model. The estimated one-factor model did not present a gasoline price below the critical value in any of the simulations, and its incremental value from fuel flexibility is, as we would expect, negligible.

4. DISCUSSION

By affecting the forecasted benefits from additional battery capacity, the chosen model of gasoline prices affects the price at which the 6.7 kWh PHEV20 battery would pay for itself. Considering uncertainty and fuel flexibility can increase the break-even battery price by almost 15% in the non-stationary two-factor model, but a discounted cash flow analysis is an adequate approximation for battery value in the other two price models. Kalhammer et al. (2007) forecast that if production of lithium-ion batteries reaches 20,000 units per year, costs for a 7 kWh PHEV20 battery could decline to $600-$750/kWh, and if production reaches 100,000 units per year, they forecast that costs could reach $400-$575/
Figure 3. Contribution of Uncertainty and Fuel Flexibility to the Estimated PHEV Value

(a) One-factor model

(b) Non-stationary two-factor model

kWh. Costs would need to be in this lower range and the purchaser would need a low discount rate for the PHEV to be economical in the one-factor model; they would need to be in this lower range and the purchaser would need to consider uncertainty and fuel flexibility while using a low discount rate for the PHEV to be economical in the non-stationary two-factor model; and discount rates above 5%/year or use of the non-stationary two-factor model would require battery prices below the low end of the forecast range. Battery prices must fall to near $200/kWh to make battery cost-effectiveness robust to the gasoline price model and to the discount rate (price of risk). Higher electricity prices or lower gasoline prices would increase the incremental value of fuel flexibility by increasing the
Figure 3. Contribution of Uncertainty and Fuel Flexibility (continued)

The break-even battery cost $BC_{\text{p}\text{e}V20}$ for a PHEV20 relative to a comparable hybrid electric vehicle using each of the three gasoline price models with parameters as in Tables 2 and 3. Shows the value in a deterministic world, the incremental value when gasoline prices are uncertain and the owner lacks fueling flexibility, and the incremental value when the owner has fueling flexibility in a world of uncertain gasoline prices. The incremental value from fuel flexibility in the one-factor model and from uncertainty in the stationary two-factor model are each too small to appear on the plot.

Figure 4. Days of Forgone Charging Across 1000 Vehicle Lifetimes

Number of days in each of 1000 simulated vehicle lifetimes that the PHEV owner would prefer to forgo charging from the grid in favor of using gasoline. The gasoline price at which the PHEV owner is indifferent between fuel sources is $1.21/gallon.
probability of gasoline prices below the critical value of fuel indifference, but they would decrease the total value of the PHEV by reducing the savings from using electricity instead of gasoline.

Considering fuel flexibility would not significantly affect the results of Kammen et al. (2009), which suggest that subsidizing PHEVs in order to obtain greenhouse gas abatement is a costly climate strategy. However, we also need to consider a real options approach to valuing PHEVs' role in climate strategy. PHEVs provide the ability to eventually constrain transportation sector emissions through future electricity sector regulations, and they may reduce the cost of generating low-carbon electricity if the vehicles' charging can be controlled to match intermittent renewable energy sources. Embedding PHEVs in the future vehicle fleet by subsidizing their purchase today is like replacing the currently held option to reduce greenhouse gases in a future economy containing a liquid-fueled vehicle fleet with the option to reduce greenhouse gases in a future economy containing a vehicle fleet with more PHEVs. The value of this new option would depend on the degree to which future climate policy is cheaper with a fleet of PHEVs and on the probability of future stringent climate policies.

The real options approach to valuing battery capacity differs in many ways from how vehicle purchasers may actually value PHEVs. The following is a partial list of omitted factors, along with an indication in parentheses of whether including the omitted factor is likely to increase the attractiveness of PHEVs to vehicle purchasers:

- Non-financial values as described in Heffner et al. (2007) (probably +)
- The possibility of needing battery replacement (-)
- Constraints on charging induced by battery chemistry or vehicle usage patterns (-)
- Dynamic electricity prices, which can include uncertainty (+, if the uncertainty is symmetric), the possibility of special off-peak rates (+), increasing rates over time (-), and the possibility of climate policy raising gasoline and electricity prices (ambiguous)
- Extreme gasoline price movements (probably +)
- The possibility of recovering more value from the battery due to selling electricity back to the grid as described by Kempton and Tomić (2005a,b) and of using the battery for stationary applications after vehicle service (+)
- The timing of the investment in the PHEV efficiency technology, with particular connection to the theory of investment timing with multiple mutually exclusive investment strategies (Dixit, 1993; Fleten et al., 2007) (-, but with failure to purchase possibly just representing a delayed purchase)
- Failure of vehicle purchasers to calculate fuel savings (Turrentine and Kurani, 2007, but see Espey and Nair, 2005) (ambiguous; note that fleet purchasers may do these calculations)
If PHEV purchasers do fail to consider the incremental value from fuel flexibility, and if fuel price volatility gives this flexibility substantial value, companies may be able to speed the transition to PHEVs and capture part of the fuel flexibility value by monetizing the fuel savings. This may also work if vehicle purchasers use higher prices for risk than the companies do. Because of the gap between the value that vehicle purchasers may attach to the options purchased by the battery capacity and the value that a company using a real options approach may attach to the battery's implicit options, it may be possible for a company to pay the PHEV buyer for a share of the options' payoffs and for this payment to be worth more to the PHEV buyer than the forgone future fuel savings would be. Because undervaluing PHEVs would leave value on the table, there may be a way for a company to give the vehicle purchaser part of this left-out value while capturing enough of the value for itself to make the arrangement worth pursuing.

One such business model could work as follows. A company could pay the PHEV owner up front upon the purchase of a PHEV in exchange for taxing each future kilowatt-hour charged from the grid. If this "tax" is linked to the local price of gasoline and specified as a percentage of the fuel cost savings, then the PHEV owner would retain the correct incentive to charge from the grid when electricity provides cheaper transportation. Meanwhile, the company would have purchased a percentage of the PHEV options' payoffs and so a percentage of the PHEV options' value. The company could earn a positive return by setting the up-front payment to the vehicle purchaser to be less than the PHEV option bundle's value, and the vehicle purchaser may want to accept the deal if the up-front payment is more than the discounted cash flow value of the stream of "taxes." As a result, both parties would value the PHEV more highly, and the vehicle buyer might be more likely to purchase it. Further work could explore the implications of this scheme, as well as the implications of the general change from gasoline pricing and payment patterns to electricity pricing and payment patterns, in behavioral models such as the mental accounting model of Prelec and Loewenstein (1998).

5. CONCLUSION

The widespread adoption of PHEVs would call not just for new ways of managing the interface between vehicles and the electric grid but also for new ways of valuing the fuel savings and emission reductions from different vehicle technologies. Discounted cash flow analyses are well-suited for evaluating improvements in traditional gasoline or diesel technologies but are less apt for evaluating multi-fuel technologies. PHEV owners, for instance, have the option to charge from the grid or to run their vehicles in gasoline-fueled charge-sustaining

14. To be acceptable to the company, such an arrangement may require a hedging strategy to offset its risks. If a perfect hedging strategy for PHEV options were to be found, the company's profit would in fact be riskless and the company would be an arbitrageur. A perfect hedging strategy would also provide new ways of valuing PHEVs' battery capacity.
mode. Because gasoline prices cannot be perfectly predicted ahead of time, each day’s option to charge creates nonlinear payoffs that are properly evaluated in a real options framework. We treat the purchase of a PHEV’s battery capacity as the purchase of a strip of European call options in which the payoffs are the fuel savings in dollars per mile and each option corresponds to the decision on some day \( d \) about whether to use grid electricity for some mile \( m \) of PHEV travel. This real options approach increases the battery price at which a PHEV’s additional battery capacity may pay for itself, but our specific valuation application shows that for the battery to actually pay for itself, one may need to believe that particular price models describe the future much better than do other plausible ones or that battery prices will fall by more than some predict near-term mass production alone will achieve. Further work could apply this real options approach to other vehicle technologies, to other vehicle markets, and to other price models.

If vehicle purchasers fail to fully value the fuel flexibility offered by multi-fuel vehicles such as PHEVs, then new business models for monetizing fuel savings may be profitable to both the vehicle purchaser and the vehicle seller and could make multi-fuel vehicles more attractive to both parties. Increasing the percentage of PHEVs in the vehicle fleet could provide direct reductions in urban air pollutants, petroleum use, and greenhouse gas emissions, and it should also lower the cost of obtaining future reductions in greenhouse gas emissions. Assessments of the cost-effectiveness of promoting PHEVs to achieve these benefits should consider how a real options approach would affect their conclusions. Future analyses could assess the influence of climate policies on the incentives to purchase PHEVs by representing these policies’ anticipated effects in the fuel price processes. Most importantly, they should consider PHEVs’ larger role in greenhouse gas abatement efforts by using a real options framework to value the abatement flexibility that society would gain from widespread adoption of PHEVs.
APPENDIX 1: CLOSED-FORM SOLUTIONS FOR PHEV OPTIONS WITH DETERMINISTIC ELECTRICITY PRICES

We value PHEV options according to the three models of the retail gasoline price process described section 3. Each of these option formulas replaces the maximization terms in equations (2) and (3) under the assumption of deterministic electricity prices. Unless the sources of risk can be replicated by traded assets, we cannot deduce unique derivative prices using only no-arbitrage arguments. We use the price of risk \( \lambda \) in order to define the risk-adjusted dynamics of the log spot prices. The market price of risk determines the market value of the PHEV's fuel flexibility, but individuals or corporations may use their own appetites for risk to derive their own valuations of the fuel flexibility (Eydeland and Wolyniec, 2003: 437). We adapt some pricing formulas and extend others in new ways. Significantly, whereas the literature exploring these price processes has treated the price of risk as a constant reduction in the risk-adjusted drift relative to the drift in the real-world price process, we reserve the label "price of risk" for the excess returns per unit of volatility. Since \( \lambda \) is the price of risk, the constant reduction in the risk-adjusted drift is therefore \( \lambda \sigma \).

First, let retail gasoline prices evolve according to the one-factor model of Schwartz (1997) given in equation (7). If gasoline were liquidly traded on a spot market or the risk were perfectly replicable, we could use the conventional Black-Scholes-Merton formula because the price would grow by \( r \) %/year in a risk-free world (Grundy, 1991; Lo and Wang, 1995). However, in an incomplete market setting, the risk-adjusted price process is more complicated:

\[
d\ln P_g(t) = \kappa(\alpha^* - \ln P_g(t))dt + \sigma dz^*
\]

where \( dz^* \) an increment of Brownian motion under the risk-adjusted probability measure and where the long-term risk-adjusted mean is

\[
\alpha^* = \alpha - \frac{\sigma\lambda}{\kappa}.
\]

The real-world drift is reduced by \( \sigma\lambda \). The mean \( \mu^*(t) \) and variance \( (\sigma^*)^2(t) \) for \( \ln P_g(t) \) under the risk-adjusted probability measure are:

\[
\mu^* = e^{-\kappa t} \ln P_g(0) + (1 - e^{-\kappa t})\alpha^* \quad (17)
\]

\[
(\sigma^*)^2 = \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa t}) \quad (18)
\]

We can find \( C'(0) \), the value of the discounted maximization term in (2) and (3), by taking discounted expectations of the nonlinear payoff. The result is:
\[ C^*(0) = e^{-rT} \left( \nu \Phi \left[ \sigma^* - \frac{\omega}{\sigma^*} \right] - K_Y \Phi \left[ -\frac{\omega}{\sigma^*} \right] \right), \] where \( \Phi(\cdot) \) is the cumulative distribution function for the standard normal distribution and \( T \) is the time of the option's expiration. In the PHEV example, \( T=d \) with \( r \) expressed as a daily rate. The PHEV option value \( C_d(0) \) follows immediately from \( C^*(0) \).

The second model of retail gasoline price changes is the non-stationary two-factor model of Schwartz and Smith (2000). As described in equations (8) through (11), it has mean-reverting short-term deviations from a long-term mean that evolves according to geometric Brownian motion. Schwartz and Smith provide formulas for pricing options on futures whose underlying asset follows this two-factor process. If there is no arbitrage between spot and futures markets, then the futures prices should converge to the spot price as the time to maturity decreases. Thus we can obtain the price of European call options on spot prices by making the option's futures contract expire on the same day the option expires. The risk-adjusted processes are:

\[ d \chi_t = \kappa \left( -\chi_t - \frac{\sigma_{\chi} \lambda_{\chi}}{\kappa} \right) dt + \sigma_{\chi} d\xi_{\chi}^* \] (22)

\[ d \xi_t = \left( \mu_{\xi} - \sigma_{\xi} \lambda_{\xi} \right) dt + \sigma_{\xi} d\xi_{\xi}^* \] (23)

\( \lambda_{\chi} \) is the price of risk in the long-term mean's process, and \( \lambda_{\xi} \) is the price of risk in the short-term deviations. We have:

\[ C^*(0) = e^{-rT} \left( F_T \Phi[d_0] - K_Y \Phi[d_0 - \sigma_0(T)] \right) \] (24)

where

\[ d_0 = \frac{\ln(F_T / K_Y) + \frac{1}{2} \sigma_0^2(T)}{\sigma_0(T)}. \]

\( F_T \) is the price at time \( t = 0 \) of the futures contract expiring at time \( T \):
Valuing Plug-in Hybrid Electric Vehicles’ Battery Capacity

\[ F_T = \exp\left[ e^{-\kappa T} x_0 + \xi_0 + A(T) \right] \]  

(25)

where:

\[ A(T) = (\mu_\xi - \sigma_\xi \lambda_\xi)T - (1 - e^{-\kappa T}) \frac{\sigma_\xi \lambda_\xi}{\kappa} \]

\[ + \frac{1}{2} \left( (1 - e^{-2\kappa T}) \frac{\sigma_\xi^2}{2\kappa} + \sigma_\xi^2 T + 2(1 - e^{-\kappa T}) \frac{\rho_\chi_\xi \sigma_\chi \sigma_\xi}{\kappa} \right) \]  

(26)

\( \sigma_0^2(T) \) is the variance of the normally distributed log of the price at the maturity of the option (i.e., at time \( T \) in our case) on the futures contract that expires at time \( T \):

\[ \sigma_0^2(T) = (1 - e^{-2\kappa T}) \frac{\sigma_\xi^2}{2\kappa} + \sigma_\xi^2 T \]  

(27)

Our third model of retail gasoline price changes is the stationary two-factor model of Korn (2005) contained in equations (9), (11), (13), and (14). This model includes mean-reverting short-term variations and mean-reverting long-term variations. Korn has made available on request an earlier version of his paper in which he includes formulas for European call options on futures. The risk-adjusted processes are:

\[ d \chi_t = \kappa \left( -\chi_t - \frac{\sigma_\chi \lambda_\chi}{\kappa} \right) dt + \sigma_\chi d z^*_\chi \]  

(28)

\[ d \xi_t = \gamma \left( \Theta - \xi_t - \frac{\sigma_\xi \lambda_\xi}{\gamma} \right) dt + \sigma_\xi d z^*_\xi \]  

(29)

Letting each futures contract expire on the same day on which its associated option expires, we obtain the following equations for use in equation (24):

\[ F_T = \exp\left[ e^{-\kappa T} x_0 + \frac{\kappa}{\kappa - \gamma} e^{-\gamma T} \xi_0 + A(T) \right] \]  

(30)

where:

\[ A(T) = -(1 - e^{-\kappa T}) \frac{\sigma_\chi \lambda_\chi}{\kappa} + \kappa \left[ \left( 1 - e^{-\gamma T} \frac{\Theta - \sigma_\xi \lambda_\xi}{\gamma} \right) \right] - \frac{\gamma \Theta}{\kappa - \gamma} \]

\[ + (1 - e^{-2\kappa T}) \frac{\sigma_\xi^2}{4\kappa} + \frac{\kappa^2}{(\kappa - \gamma)^2} (1 - e^{-2\gamma T}) \frac{\sigma_\xi^2}{4\gamma^2} \]

\[ + \frac{\kappa}{(\kappa - \gamma)} \left( 1 - e^{-(\kappa + \gamma) T} \right) \frac{\rho_\chi_\xi \sigma_\chi \sigma_\xi}{(\kappa + \gamma)} \]  

(31)
and where:

\[
\sigma_0^2(T) = (1-e^{-2\kappa T}) \frac{\sigma_x^2}{2\kappa} + (1-e^{-2\gamma T}) \frac{\kappa^2}{(\kappa-\gamma)^2} \frac{\sigma_z^2}{2\gamma} + 2(1-e^{-(\kappa+\gamma)T}) \frac{\kappa}{(\kappa-\gamma)} \frac{\rho_x \sigma_x \sigma_z \sigma_{\varepsilon}}{(\kappa+\gamma)}
\]

(32)

APPENDIX 2: CLOSED-FORM SOLUTION FOR THE SPREAD CALL OPTION WITH STOCHASTIC ELECTRICITY PRICES

Here we present the pricing formula for the spread call option in the absence of arbitrage. The spread call option is used to value PHEVs when there is real-time pricing of electricity or, indeed, to value any dual-fuel vehicle with fuel prices following geometric mean-reverting processes. The simplest pricing technique is to model the evolution of the spread directly, but as Carmona and Durrleman (2003) and Eydeland and Wolyniec (2003) note, the assumptions required by this approach are probably not valid in energy markets. Deng et al. (2001) price spark spread options in the case where the futures prices are mean-reverting, but we use the analytical solution of Tsitakis et al. (2006) because they assume only that the spot prices of the underlying assets follow mean-reverting exponential Ornstein-Uhlenbeck processes, that each commodity has a price of risk, and that there is no arbitrage. However, they only consider the case in which the long-term mean of each logarithmic price process is zero. We adapt their solution to include a non-zero long-term mean and show only the steps in the proof that change as a result. Also, we correct three typographical errors in the original presentation by using \( f_j \) instead of \( f_i \) in the equation for \( f_j \) and by including a negative sign in front of the integrals in \( f_3 \) and \( f_4 \).

Let the gasoline spot price have index \( i=1 \) and the electricity spot price have index \( i=2 \). Also, let the price processes follow geometric mean-reverting processes:

\[
d(\ln P_i(t)) = \kappa_i(t)[\alpha_i(t) - \ln P_i(t)] dt + \sum_{j=1}^{2} \sigma_{ij}(t) dB_j(t)
\]

(33)

where \( P_i(t) \) is the spot price of commodity \( i \) at time \( t \), \( \kappa_i(t) \) and \( \sigma_i(t) \) are the speed of mean reversion and volatility parameters as in (7), and \( B_i(t) \) and \( B_j(t) \) are independent Brownian motions. As in (7), \( \alpha_i(t) \) may be interpreted as providing at time \( t \) the long-term mean of the log price process for commodity \( i \). \( \alpha_i(t) \) was implicitly equal to zero in Tsitakis et al. (2006). We want to price the European call option with the following payoff at maturity \( T \):

\[
\max \left( R_1(T) - K_H P_2(T), 0 \right)
\]

(34)
Now let $\theta_i(t)$ be the price of risk for commodity $i$ so that $dW_i(t) = dB_i(t) - \theta_i(t) \, dt$ is a Brownian motion under the equivalent measure $Q$. The risk-adjusted process becomes:

$$
\begin{align*}
\frac{d}{dt} \ln(P_1(t)) &= [\kappa_i(t) - \ln(P_1(t))] \\
&+ \frac{2}{\sum_{j=1}^{2} \sigma_{ij}(t) \theta_j(t)] dt + \frac{2}{\sum_{j=1}^{2} \sigma_{ij}(t)} dW_j(t) \\
&= \sum_{j=1}^{2} \sigma_{ij}(t) \theta_j(t) dt + dW_j(t) \\
\end{align*}
$$

(35)

With this risk-adjusted process, we solve the following two equations to get $f_3$ and $f_4$:

$$
\frac{df_3}{dt} + f_1 \kappa_1 + \frac{2}{\sum_{j=1}^{2} \sigma_{ij} \theta_j f_1} = 0
$$

(36)

$$
\frac{df_4}{dt} + f_2 \kappa_2 + \frac{2}{\sum_{j=1}^{2} \sigma_{ij} \theta_j f_2} = 0
$$

(37)

The value of the European spread call option at time $t$ with maturity $T$ and “heat rate” $K_H$ is:

$$
C(P_1, P_2, t) = \frac{1}{2} \exp(A) \exp(B) \text{erfc}(E) - \frac{K_H}{2} \exp(A) \exp(B) \exp(\Gamma) \exp(\Delta) \text{erfc}(E + \sqrt{\Delta})
$$

(38)

where $\text{erfc}$ is the complementary error function and where:

$$
A = f_1(t) \ln(P_1) + f_3(t)
$$

$$
B = \int_{T}^{t} \left( \frac{v_{11}(s)f_1^2(s)}{2} - r \right) ds
$$

$$
\Gamma = \int_{T}^{t} \left( v_{11}(s)f_1^2(s) - v_{12}(s)f_1(s)f_2(s) \right) ds
$$

$$
\Delta = \int_{T}^{t} \left( v_{11}(s)f_1^2(s) - 2v_{12}(s)f_1(s)f_2(s) + v_{22}(s)f_2^2(s) \right) ds
$$
\[ E = \frac{\Gamma - \ln\left(\frac{1}{K}\right)}{2\sqrt{\Delta}} \]

\[ f_1(t) = \exp\left(\int_{T}^{t} \kappa_1(s) \, ds\right) \]

\[ f_2(t) = \exp\left(\int_{T}^{t} \kappa_2(s) \, ds\right) \]

\[ f_3(t) = -\int_{T}^{t} f_1(s) \sum_{j=1}^{2} \sigma_{1j}(s) \theta_j(s) \, ds - \int_{T}^{t} f_1(s) \kappa_1(s) \alpha_1(s) \, ds \]

\[ f_4(t) = -\int_{T}^{t} f_2(s) \sum_{j=1}^{2} \sigma_{2j}(s) \theta_j(s) \, ds - \int_{T}^{t} f_2(s) \kappa_2(s) \alpha_2(s) \, ds \]

\[ v_{11} = \sigma_{11}^2 + \sigma_{12}^2 \]

\[ v_{12} = \sigma_{11} \sigma_{21} + \sigma_{12} \sigma_{22} \]

\[ v_{22} = \sigma_{21}^2 + \sigma_{22}^2 \]

For the PHEV option valuation, we have \( t=0 \) at the time of purchase and \( T = d \), the day the charging might take place.

ACKNOWLEDGMENTS

Thanks to Steven Evans, John Stanley, Sintana Vergara, and especially Daniel Kammen for their comments and support. Thanks also to the two anonymous reviewers for their insights and comments and to Stelios Xanthopoulos for corresponding regarding his spread call option formula. This material is based upon work supported by a National Science Foundation Graduate Research Fellowship, and an early version received the 2008 Dennis J. O'Brien USAEE/IAEE Best Student Paper Award. We honor the memory of Alex Farrell and his multifaceted work towards the decarbonization of transportation.
REFERENCES


